Connections between graphs and matrix spaces

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Given a graph G, the graphical matrix space consists of matrices whose non-zero entries can only be at those positions corresponding to edges in G. In this talk, I will focus on some surprising connections between properties of graphs and matrix spaces. Specifically, such connections are found between acyclicity and nilpotency, between strong connectivity and irreducibility, and between isomorphism and conjugacy/congruence. For each connection, I will introduce three types of correspondences, namely the basic correspondence, the inherited correspondence (for subgraphs and subspaces), and the induced correspondence (for induced subgraphs and restrictions).

Some correspondences lead to intriguing generalizations of classical results, such as Dieudonné's result regarding the largest dimension of matrix spaces containing only singular matrices (*Arch. Math.*, 1948), and a celebrated theorem of Gerstenhaber regarding the largest dimension of nil matrix spaces (*Amer. J. Math.*, 1958). In this context, I will also present an open problem motivated by a well-known theorem of Atkinson, who proved a tight upper bound on the dimensions of matrix spaces in which every matrix has at most k non-zero eigenvalues (*Bull. Lond. Math. Soc.*, 1980).

For more details, please refer to our preprint [1].

[1] Yinan Li, Youming Qiao, Avi Wigderson, Yuval Wigderson, and Chuanqi Zhang. More connections between graphs and matrix spaces. arXiv:2206.04815, 2022.