The number of colored binary partitions as a sum of three squares

Bartosz Sobolewski, Maciej Ulas

Jagiellonian University

Let $b_m(n)$ denote the number of partitions of a non-negative integer n into powers of 2, where each part can take one of m colors. For $m = 1$ this is the classical binary partition function, already considered by Euler, while the study for general m has been initiated recently. In the talk we focus on the Diophantine equation

$$b_m(n) = x^2 + y^2 + z^2.$$

In the case $m = 2^k - 1$ we provide a (suprisingly simple) characterization of the set S_m, consisting of n such that a solution exists. As an application, we derive a precise estimate for the counting function of S_{2^k-1}. We also discuss possible directions of further research, such as the case of general $m \geq 1$, as well as numerical results concerning related equations $b_1(n) = x^2 + y^2 + z^4$ and $b_1(n) = x^2 + y^2$.

The talk is based on joint work with Maciej Ulas.