The stochastic sandpile model on complete graphs

Thomas Selig

Xi'an Jiaotong-Liverpool University, Suzhou, China

The stochastic sandpile model (SSM) is a generalisation of the standard Abelian sandpile model (ASM), in which topplings of unstable vertices are made random. When unstable, a vertex sends one grain to each of its neighbours independently with probability $p \in (0, 1)$. We study the SSM on complete graphs. Our main result is a description of the recurrent states of the model, i.e. those that appear infinitely often in its long-time running. We show that these are given by convex sums of recurrent states for the ASM. This allows us to recover a well-known result: that the number of integer lattice points in the n-dimensional permutation polytope is equal to the number of labeled spanning forests on n vertices. We then study a family of so-called *partial* SSMs, in which some vertices topple randomly, while others topple deterministically (as in the ASM, sending one grain to all neighbours). We show that this distinction is meaningful, yielding sets of recurrent states that are in general different from those of both the ASM and SSM. We also show that to get all recurrent states of the SSM, we can allow up to two vertices to topple deterministically.