

Probabilistic intuition and small subgraph games

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Let H be a fixed graph and consider the following two-player game on the edges of K_n , the complete graph with n vertices: starting with an empty graph G on the vertex set of K_n , in each round the first player chooses b edges from K_n which have not previously been chosen, and the second player immediately and irrevocably picks one of these edges and adds it to G . How large can b be such that the first player has a strategy which ensures that H appears in G , regardless of how the second player acts? A heuristic argument known as the *probabilistic intuition* suggests that this value roughly coincides with the one for which H appears with probability at most $1/2$ if both players play completely at random. Resolving a conjecture by Bednarska-Bzdęga, Hefetz, and Łuczak, we show that the probabilistic intuition is indeed correct in this case. (Note: The proof itself requires some probabilistic intuition as well!)