

Three-dimensional maps and subgroup growth

Rémi Bottinelli, Laura Ciobanu, Alexander Kolpakov

University of Neuchâtel, Switzerland

Maps on surfaces were studied by Tutte, Jones, Singerman, among many others. Pavings, that are three-dimensional analogues of maps, were introduced by Arquès and Koch, and further studied by Lienhardt. In a series of works, Mednykh and Nedela solved Tutte's enumeration problem for maps on surfaces of given genus, and regardless of genus (with Breda d'Azevedo).

In this talk I will show how to derive a generating series for the number of pavings on n darts, thus solving an analogue of Tutte's problem in dimension three (unconstrained to the topological type of the resulting cellular complex). The generating series thus derived also counts free subgroups of given index in the triple free product of order 2 cyclic groups, linking to the work on subgroup growth by Lubotzky, Mednykh, Müller and Schlage-Puchta.

Computer experiments performed with software designed by the authors provide some statistics about the topology and combinatorics of pavings on $n \leq 16$ darts.