Critical Problem for a q-analogue of polymatroids

IMAMURA, Koji and SHIROMOTO, Keisuke

Kumamoto University, Japan

The *Critical Problem* was posed by H. Crapo and G.-C. Rota in 1970 to formulate various problems in extremal combinatorial theory. Let $\mathbb{F}_q$ be a finite field of q elements. For any subset $S \subseteq \mathbb{F}_q^k$, define the *critical exponent* of S as follows:

$$c(S, q) := k - \max\{\dim D : D \leq \mathbb{F}_q^k \text{ and } D \cap S = \emptyset\}.$$

The problem is to find the critical exponent for a given matroid with the ground set S.

They also proved another approach to the Critical Problem, celebrated as the *Critical Theorem*. Let $p(M; \lambda)$ denote the characteristic polynomial of a matroid M and let M/X denote the contraction of M by $X \subseteq E$. Then the theorem is described as follows:

Theorem 1 *Let C be a k-dimensional subspace of $\mathbb{F}_q^n$ and set $E := \{1, \ldots, n\}$. For any $X \subseteq E$ and any $m \in \mathbb{Z}^+$, the number of ordered m-tuples $(\boldsymbol{v}_1, \ldots, \boldsymbol{v}_m)$ of vectors in C with* $\mathrm{supp}(\boldsymbol{v}_1) \cup \cdots \cup \mathrm{supp}(\boldsymbol{v}_m) = X$ *is $p(M_C/(E - X); q^m)$, where* $\mathrm{supp}(\boldsymbol{v}) = \{i \in E \mid v_i \neq 0\}$ *for $\boldsymbol{v} = (v_1, \ldots, v_n) \in \mathbb{F}_q^n$.*

In this talk, we study the Critical Problem for a q-analogue of *polymatroids*, one of the generalizations of matroids. Associating them with a *Delsarte rank-metric code*, an $\mathbb{F}_q$-subspace of a matrix space over $\mathbb{F}_q$ equipped with the rank metric, we prove a q-analogue version of the Critical Theorem.