Block avoiding sequencings of Steiner systems

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A *partial* (n,k,t)-*Steiner system* is a pair $(X,\mathcal{B})$ where X is an n-set of *vertices* and $\mathcal{B}$ is a collection of k-subsets of X called *blocks* such that each t-set of vertices is a subset of at most one block. An ℓ-*block avoiding sequencing* of such a system is a labelling of its vertices with distinct elements of $\{0,\dots,n-1\}$ such that no block is contained in a set of ℓ vertices with consecutive labels. This talk will discuss block avoiding point sequencings of partial Steiner systems. In particular, we outline a proof that, for fixed k and t, any partial (n,k,t)-Steiner system has an ℓ-good sequencing for some $\ell=\Theta(n^{1/t})$ as n becomes large. This result is perhaps of most interest in the case $k=t+1$ where results of Kostochka, Mubayi and Verstraëte show that the value of ℓ cannot be increased beyond $\Theta((n\log n)^{1/t})$. A special case of this result shows that every partial Steiner triple system (partial $(n,3,2)$-Steiner system) has an ℓ-block avoiding sequencing for each positive integer $\ell \le 0.0908\,n^{1/2}$. This is joint work with Padraig Ó Catháin.