Product structure of graph classes with bounded treewidth

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This talk will introduce the topic of graph product structure theory. I will show that many graphs with bounded treewidth can be described as subgraphs of the strong product of a graph with smaller treewidth and a bounded-size complete graph. To this end, define the *underlying treewidth* of a graph class $\mathcal{G}$ to be the minimum non-negative integer c such that, for some function f, for every graph $G \in \mathcal{G}$ there is a graph H with $\mathrm{tw}(H) \leqslant c$ such that G is isomorphic to a subgraph of $H \boxtimes K_{f(\mathrm{tw}(G))}$. I'll introduce *disjointed partitions* of graphs and show they determine the underlying treewidth of any graph class. Using this result, I will show that the class of planar graphs has underlying treewidth 3; the class of $K_{s,t}$-minor-free graphs has underlying treewidth s (for $t \geqslant \max\{s, 3\}$); and the class of K_t-minor-free graphs has underlying treewidth $t-2$. This is joint work with Rutger Campbell, Katie Clinch, Marc Distel, Pascal Gollin, Kevin Hendrey, Tony Huynh, Freddie Illingworth, Youri Tamitegama, Jane Tan and David Wood [https://arxiv.org/abs/2206.02395].