Cycles of length 3 and 4 in edge-colored complete graphs with restrictions in the color transitions

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A walk in an edge-colored walk is said to be a properly colored walk iff every two consecutive edges have different color, this includes the first and last edges when the walk is closed. Properly colored walks have shown to be an effective way to model certain real applications in different fields. In view of this, it is natural to ask about the existence of properly colored walks with restrictions in the transitions of colors allowed in the edges of a graph. Let H be a graph possibly with loops and G a graph. We say that G is an H-colored graph whenever there exists a function $c : E(G) \longrightarrow V(H)$. A path $(v_1, \cdots, v_n)$ in an H-colored graph G is an H-path iff $(c(v_1v_2), c(v_2v_3), \cdots, c(v_{n-1}v_n))$ is a walk in H, and a cycle $(v_1, \cdots, v_n, v_1)$ is an H-cycle whenever $(c(v_1v_2), c(v_2v_3), \cdots, c(v_{n-1}v_n), c(v_nv_1), c(v_1v_2))$ is a walk in H. Hence, H decides which color transitions are allowed in a walk in order to be an H-walk.

Let G be an H-colored complete graph. In this talk, we show conditions implying that each vertex of G is contained in an H-cycle of length 3 (respectively 4).