Beyond the log-concavity of the restricted partition function

Krystian Gajdzica

Jagiellonian University

The talk is devoted to the so-called log-behaviour of the restricted partition function $p_{\mathcal{A}}(n,k)$ — that is the number of partitions of n with parts in the multiset $\{a_1, a_2, \ldots, a_k\}$, where $\mathcal{A} = (a_i)_{i=1}^{\infty}$ is a weakly increasing sequence of positive integers, and $k \in \mathbb{N}_+$ is fixed.

It turns out that for a given parameter k the sequence $(p_{\mathcal{A}}(n,k))_{n=1}^{\infty}$ is usually (but not always) log-concave for sufficiently large values of n. In other terms, the inequality

$$p_{\mathcal{A}}^2(n,k) > p_{\mathcal{A}}(n+1,k)p_{\mathcal{A}}(n-1,k)$$

holds for all large numbers n. During the talk we will present an effective criterion for the above property, and investigate some of its generalizations. At first, we will deal with some basic extensions such as, for instance, the strong log-concavity of $p_{\mathcal{A}}(n,k)$ — in that case, the corresponding inequality takes the form:

$$p_{\mathcal{A}}^2(n,k) > p_{\mathcal{A}}(n+m,k)p_{\mathcal{A}}(n-m,k),$$

where $m \in \mathbb{N}_+$ is arbitrary. Afterward, we will go to the main part of the talk, and focus on both the higher order Turán inequalities and the r-log-concavity problems for the function $p_{\mathcal{A}}(n,k)$.