Cycle lengths of 1-factorisations of complete graphs and complete bipartite graphs

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A 1-factor of a graph G is a subset M of the edges of G, such that every vertex is incident to exactly one edge in M. A 1-factorisation F of G is a partition of the edges of G into 1-factors. Each pair of 1-factors in F induces a subgraph of G, which is the union of even cycles. If every cycle induced by F is Hamiltonian then we call F a perfect 1-factorisation of G. It has been conjectured that if G is the complete graph K_{n+1} or the complete bipartite graph $K_{n,n}$ for an odd integer n then a perfect 1-factorisation of G exists. This conjecture is a long way off being resolved. We study a weak approximation of perfect 1-factorisations by using Latin squares to construct a class of 1-factorisations of complete graphs, and a class of 1-factorisations of complete bipartite graphs, which induce no short cycles. We also construct a family of Latin squares which are devoid of 2×2 Latin subsquares.